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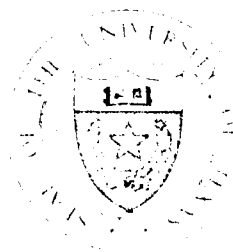
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A "MULTIPLE PIVOTING" ALGORITHM FOR GOAL-INTERVAL PROGRAMMING FORMULATIONS

by

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ABSTRACT

Goal programming has become a popular and important planning tool in many areas. However, the main direction of goal programming research has been in formulating models instead of seeking procedures that would provide improved computational efficiency. A recent paper by Charnes, Cooper, Klingman, and Niehaus has exploited the specialized structures of a class of convex goal programming problems to obtain increases in efficiency. In this paper, an analytical formulation for a piecewise linear goal-interval programming problem is presented and a special purpose algorithm is developed. The algorithm may combine several standard simplex pivots into a single pivot through the use of an extended minimum ratio test.

1. Introduction

For a substantial period of time following the development of
the initial goal programming model for a scheme of executive compensation
by Charnes, Cooper, and Ferguson [11], the main emphasis of goal programming
was directed towards exploiting this tool in a wide variety of applications
and developing formal statistical theory. In the last few years, there
have been considerable advances in the "state of the art." With the
development of the field of public management science, more sophisticated
models have been needed to solve complex problems of manpower and resource
planning. One of the most important new models joins goal programming and
Markov processes. This model and many other current advances can be found
in the Office of Civilian Manpower Management, U. S. Navy, (OCMM) series
[14].

In spite of the wide use of goal programming, only a few results
have been obtained that will secure goal programming algorithms that are
computationally efficient. A paper by Charnes, Cooper, Klingman, and
Niehaus [12] provides a specialized solution technique by a transformation
to a specialized linear interval programming problem for a class of "goal
functionals." It is also widely known that the problem of minimizing the sum
of the absolute values of the goal deviations, the L_1 Norm problem, can be
placed in a more tractable format by reduction to a linear programming
equivalent. In the case of solving for the Least Absolute Value Estimators
(LAVE's), the special structure of the problem has enabled the creation of
special purpose primal linear programming codes [1, 2, 4, 5, 16].

The difficulties presented by nonlinear convex goal functionals
can be resolved when the functional is separable. In this case, the functional
can be approximated by a convex piecewise linear functional to any degree of

accuracy that may be desired by using the techniques of Charnes and Lemke [14]. The piecewise linear functional approach was used by Charnes and Collomb [8] for models of optimal economic stabilization policy, but instead of using the classical goal programming formulation they utilized the concept of a goal-interval for their models. Thus, a goal-interval functional, in a single variable x, will assume the same minimum value over an interval $[g^-, g^+]$ as opposed to a goal programming problem which assumes its minimum value at a unique point. Collomb [15] also studied goal-interval models for decentralization in management and intertemporal analysis. Charnes and Cooper [9] found that if the usual goal programming formulations were applied to assist in resource allocation decisions for the U. S. Coast Guard's Marine Environmental Protection (MEP) program, then the results were less than satisfactory. Therefore, to obtain more flexibility in the models, they employed the goal-interval approach and obtained results more in line with the desires of the policy makers.

The conceptual flexibility provided by goal-interval programming would be negated to a certain degree if the practical implementation for computational purposes caused many difficulties. In the paper by Charnes, Cooper, Klingman, and Niehaus [12], they demonstrated that the goal-interval programs can be reduced to a linear programming equivalent. Hence, the full computational and interpretative power of linear programming is maintained. However, since there are many variables and constraints in these types of problems, it would be very advantageous in terms of money and computer time to have an efficient algorithm to solve the problems. A specialized primal simplex algorithm will be developed to exploit the convex piecewise linearity of the functional for deviations not in the goal interval. The algorithm

will have the ability to combine several standard simplex pivots into a single pivot which will enhance the computational aspect of applying such techniques.

2. Analytical Formulations of Goal-Interval Programming

A desirable aspect of goal-interval model is that it presents a mixture of a satisficing behavior and an optimization formulation. The presence of separable convex piecewise linear functional allows the convenient reduction to a linear programming equivalent. Collomb [15] pointed out that for each variable there are thresholds, which define regions as "satisfactory" or "less than satisfactory." When an optimization procedure is applied to a goal-interval model, then the optimal solution may have some variables in regions which are "less than satisfactory." Other variables in the optimal solution may be permitted to vary within the goal-interval since this provides a range of indifference in satisfaction.

Figure 1 represents a goal-interval model in terms of a single incremental variable, and it is similar to those found in Charnes and Collomb [8] and Collomb [15]. The threshold (goal) levels are given by the g_k's, and the goal-interval is $[g_k^-, g_k^+]$. The w_i^- and w_i^+ terms are the relative weights (penalties) assigned to the "less than satisfactory" segments of the functional.

At this point a particular type of problem that may arise in an area such as resource allocation is formulated. The problem for consideration is as follows:

$$\text{Minimize} \quad \sum_{k=1}^{m} w_k^- \delta_k^- + \sum_{k=1}^{m} w_k^+ \delta_k^+ \tag{1}$$

subject to

$$g_k^- - \delta_k^- \leq a_{k1}x_1 + a_{k2}x_2 + \ldots + a_{kn}x_n \leq g_k^+ + \delta_k^+ \quad k = 1, 2, \ldots, m$$

$$b_i^- \leq h_{i1}x_1 + h_{i2}x_2 + \ldots + h_{in}x_n \leq b_i^+ \quad i = 1, 2, \ldots, q$$

$$\delta_k^+, \delta_k^- \geq 0 \text{ for } k = 1, 2, \ldots, m$$

Rewritten in matrix notation, the problem becomes:

$$\text{MINIMIZE} \quad w^- \delta^- + w^+ \delta^+$$

subject to

$$g^- - \delta^- \leq Ax \leq g^+ + \delta^+$$

$$b^- \leq Hx \leq b^+$$

$$\delta^+, \delta^- \geq 0$$

The system $b^- \leq Hx \leq b^+$ is attached to represent physical constraints (limitations) that may occur and that must be satisfied before any attempt is made to resolve the goal constraints. The goal-interval for the k-th goal constraint is $[g_k^-, g_k^+]$ where $g_k^- \leq g_k^+$. For the case of $g_k^- = g_k^+$, then the k-th goal constraint can be reformulated as a usual goal equation since the goal-interval has been reduced to a unique point. If a vector $\hat{x}$ exists such that $A_k\hat{x} > g_k^+$, then $\delta_k^+ > 0$ and $A_k\hat{x} = g_k^+ + \delta_k^+$, where A_k is the k-th row of the matrix A. Thus constraint value is not in the goal-interval, and a penalty in the amount of $w_k^+\delta_k^+$ is incurred for the excess deviation associated with this constraint. A simple example for interpretation of this goal-interval

constraint may arise if the labor force is divided into classes where a particular class may be associated with a constraint such as budgeting. If the operation requires more employees in the class than is normally provided for, then additional employees must be hired, trained, and, in general, "taken care of." The penalty w_k^+ associated with the addition of δ_k^+ number of employees may arise due to interest on loans used for the expansion or due to whatever other criteria are used.

The previous problem (1) is one of the simplest formulations for a goal-interval model. The assumption of constant penalty rates for deviations outside of the goal-intervals is the main reason for this simplicity. In this paper, an extension of problem (1) is considered that has different slopes for the piecewise linear segments. This type of problem may occur is when the class of employees to be hired are technicians, and an additional allocation problem for the existing resources is created. If a large number of technicians must be hired and trained, then there are limits on the number that can be trained efficiently at a time. Thus, additional classrooms, labs, and instructors must be allocated. Larger penalties must be incurred at different goal-intervals as the various components are utilized completely. This is illustrated graphically by figure 2. Certain aspects of this formulation will be utilized during the development of the specialized algorithm for the goal-interval problem.

3. Development of Algorithm

The algorithm developed in this section is a primal simplex algorithm which will exploit the specialized structure of the linear goal-interval programming problems. Various portions of the algorithm can be recognized as modifications and extensions of other techniques. It is due to the ability to integrate these algorithms into the development that enables computational simplifications that will provide the solution procedure. Most of the algorithms and techniques employed have been mentioned in the preceding two sections. However, there is one additional algorithm to be utilized here. It was constructed by Armstrong and Hultz [3] to study restricted case of the Least Absolute Value Estimator (LAVE's)

problem. In this paper, a method is provided for a "multiple pivoting" technique upon which some extrinsic characteristics of the algorithm of this paper will rely.

The evolution of the algorithm will begin with the following goal programming problem statement:

$$\text{MINIMIZE} \quad z = \sum_{k=1}^{m} \left(\sum_{i=1}^{p(k)} w_{ki}^{+} \delta_{ki}^{+} + \sum_{i=1}^{q(k)} w_{ki}^{-} \delta_{ki}^{-} \right), \tag{3.1}$$

subject to

$$\sum_{j=1}^{n} a_{kj} x_j + \sum_{i=1}^{q(k)} \delta_{ki}^{-} - \sum_{i=1}^{p(k)} \delta_{ki}^{+} + \delta_{ko} = g_{ko}^{+}, \quad k = 1,2,\ldots,m. \tag{3.2}$$

$$0 \leq \delta_{ki}^{+} \leq g_{ki}^{+} - g_{k,i-1}^{+}, \quad k = 1, 2, \ldots, m; \ i=1,2,\ldots,p(k) \tag{3.3}$$

$$0 \leq \delta_{ki}^{-} \leq g_{k,i-1}^{-} - g_{k,i}^{-}, \quad k = 1, 2, \ldots, m; \ i=1,2,\ldots,q(k) \tag{3.4}$$

$$0 \leq \delta_{ko} \leq g_{ko}^{+} - g_{ko}^{-}, \quad k = 1, 2, \ldots, m. \tag{3.5}$$

It should be noted that (3.1)-(3.5) is a completely general representation of the formulations of the problems and models discussed in section 2. Any linear inequality constraint can be written as a linear interval constraint of the form:

$$b_i^{-} \leq \sum_{j=1}^{n} h_{ij} z_j \leq b_i^{+} \tag{3.6}$$

by letting b_i^{+} or b_i^{-} (whichever is appropriate) be $U > 0$ (or possibly $-U$), where U is a non-Archimedean transcendental for the unbounded case. It was stated in the previous section that these types of constraints were due to physical limitations on the system, and that they must be satisfied before

the attempt is made to resolve the problem of the conflicting goals. Although this type of constraint has not explicitly incorporated into the system (3.2)-(3.5), the situation can be handle by assigning an arbitrarily large positive weight, i.e., the non-Archimedean transcendental M > 0, to the deviations associated with the constraints of the form of (3.6). For simplicity, it is assumed that the system associated with (3.6) is consistent.

To demonstrate the equivalence between the goal programming problem given by (3.2)-(3.5) and the goal programming problems stated in the preceding section four lemmas are stated. The proofs for these lemmas are extensions of work found in the references, and, therefore, we shall merely indicate the appropriate reference rather than duplicate results.

LEMMA 1:

Let $(\tilde{x},\overset{\sim}{\delta})$ be an optimal solution to (3.1)-(3.5). It then follows that

$$\left(\sum_{i=1}^{p(k)} \overset{\sim}{\delta}_{ki}^{+}\right) \left(\sum_{i=1}^{q(k)} \overset{\sim}{\delta}_{ki}^{-}\right) = 0; \quad k = 1, 2, \ldots, m.$$

PROOF:

See Charnes, Cooper and Ferguson [11]. Note the assumption that

$$w_{ki} > 0.$$

LEMMA 2:

Let $(\tilde{x},\overset{\sim}{\delta})$ be an optimal solution to (3.2)-(3.5). It then follows that

$\overset{\sim}{\delta}_{ki}^{+} < g_{ki}^{+} - g_{k,i-1}^{+}$ implies $\overset{\sim}{\delta}_{k\ell}^{+} = 0$ for $\ell > i$, and $\overset{\sim}{\delta}_{ki}^{-} < g_{k,i-1}^{-} - g_{k,i}^{-}$ implies

$\overset{\sim}{\delta}_{k\ell}^{-} = 0$ for $\ell > i$.

PROOF

See Charnes and Lemke [14]. Note the assumption $w_{ki} < w_{k\ell}$ when $\ell > i$.

LEMMA 3:

Let $(\tilde{x}, \tilde{\delta})$ be an optimal solution to (3.1)-(3.5). It then follows that

$$\tilde{\delta}^+_{ki} > 0 \text{ implies } \tilde{\delta}^+_{k\ell} = g^+_{k\ell} - g^+_{k\ell -1} \text{ for } \ell < i,$$

and

$$\tilde{\delta}^-_{ki} > 0 \text{ implies } \tilde{\delta}^-_{k\ell} = g^-_{k,\ell-1} - g^-_{k,\ell} \text{ for } \ell < i,$$

PROOF:

See Charnes and Lemke [14].

LEMMA 4:

Let r be the rank of the matrix $A = (a_{ij})$. If $r = m$ then the optimal objective value for (3.1)-(3.5) is $z = 0$.

PROOF:

See Charnes, Cooper, Klingman and Niehaus [12]. In addition, they show that if $r+1=m$, then the optimal solution can be obtained explicitly.

At any stage, the algorithm will work with a submatrix of A which has full row rank and forms a basis for the vector space spanned by the rows of A. The current solution will be such that a basic constraint holds as an equality at one of the extreme points for the goal functional; that is, it will pass through g^-_{ki} or g^+_{ki} for some i whenever k is basic. Also, the conditions of lemmas 1, 2, and 3 are satisfied by the current solution. The fundamental strategy of the algorithm is as follows:

(i) Without varying r-1 of the basic constraints (where r is the rank of A), determine a direction of change for the remaining basic constraint

so as to decrease the objective function's value. If it is not possible to obtain a decrease for any of the basic constraints, then the current solution is optimal.

(ii) Assuming that the value of the objective function can be decreased, then move in the prescribed direction until the rate of change becomes nonnegative. Since the rate of change, itself, is a piecewise linear convex function, then it will have changes of slope occurring whenever movement along the constraint intersects an extreme point for the goal functionals. Therefore, a new basis will be formed by the previous (r-1) basic constraints which remained stationary, and the constraint whose value caused the rate of change of the objective function to become nonnegative.

Using matrix notation, (3.1)-(3.5) can be rewritten in the following form:

$$\text{MINIMIZE } z = w^+\delta^+ + w^-\delta^-, \tag{3.7}$$

subject to

$$Ax + I^-\delta^- - I^+\delta^+ + I\delta = g_0^+ \tag{3.8}$$

$$0 \le \delta^+ \le \Delta g^+ \tag{3.9}$$

$$0 \le \delta^- \le \Delta g^- \tag{3.10}$$

$$0 \le \delta \le \Delta g_0 \tag{3.11}$$

where all vectors and matrices are defined in accordance with the original summation representation.

The matrix A may be partitioned into $A = \binom{F}{R}$ where F is a matrix with full row rank. The algorithm begins by choosing such a matrix F (see Ben-Israel and Charnes [6] for a method to accomplish this).

The complete problem is now rewritten by partitioning it into the various components associated with the matrices F and R. The problem has the following representation:

$$\text{MINIMIZE } z = w_F^+ \delta_F^+ + w_R^+ \delta_R^+ + w_F^- \delta_F^- + w_R^- \delta_R^-$$

subject to

$$\alpha + I_F^- \delta_F^- - I_F^+ \delta_F^+ + I_F \delta_F = g_{Fo}^+$$

$$RF^{\#} \alpha + I_R^- \delta_R^- - I_R^+ \delta_R^+ + I_R \delta_R = g_{Ro}^+$$

$$0 \le \delta^+ \le \Delta g^+$$

$$0 \le \delta^- \le \Delta g^-$$

$$0 \le \delta \le \Delta g_0$$

where $\alpha = Fx$

It is assumed that, at any stage (iteration), the algorithm has a current solution which satisfies the following properties.

(i) The conditions of lemmas 1, 2, and 3 are satisfied.

(ii) All components of δ_F^-, δ_F^+ and δ_F are either at their upper or lower bound.

(iii) For each nonbasic goal constraint, exactly one representative component of δ_R^-, δ_R^+ and δ_R is between its upper and lower limit.

Property (iii) eliminates the difficulties encountered when a degenerate solution exists. These difficulties may be circumvented by a

perturbation technique similar to that described by Charnes [7]. To facilitate the algorithmic description, the possibility of degeneracy is ignored here.

An initial basic solution will clearly satisfy (i) and (ii), and, by assumption, it will also satisfy (iii). The following develops a procedure for moving to another basic solution which satisfies these properties and decreases the value of z.

At this point, additional notation that will be used throughtout the remainder of this papaer is presented. Let $\delta^*_{k,s(k)}$ be the component from δ^-_R, δ^+_R, and δ_R currently not at its upper or lower bound.

$$
\delta^*_{k,s(k)} = \begin{cases}
\delta^+_{k,\ell} & \text{when } \delta^-_{ki} = 0 \text{ for all } i, \ \delta_{ko} = 0, \\
& \qquad \delta^+_{ki} = \Delta g^+_{ki} \text{ for } i < \ell \text{ and} \\
& \qquad 0 < \delta^+_{k\ell} < g^+_{k\ell} - g^+_{k,\ell-1} = \Delta g^+_{k\ell} \\[2ex]
\delta_{ko} & \text{when } \delta^-_{ki} = \delta^+_{ki} = 0 \text{ for all } i \\
& \qquad \text{and } 0 < \delta_{ko} < g^+_{ko} - g^-_{ko} = \Delta g_{ko} \\[2ex]
\delta^-_{k\ell} & \text{when } \delta^+_{ki} = 0 \text{ for all } i, \\
& \qquad \delta_{ko} = \Delta g_{ko} \\
& \qquad \delta^-_{ki} = \Delta g^-_{ki} \text{ for } i < \ell \\
& \qquad \text{and } 0 < \delta^-_{k\ell} < g^-_{k,\ell-1} - g^-_{k,\ell} = \Delta g^-_{k\ell}
\end{cases}
$$

A notation for the indexing of various sets of also required.

$$IP = \{k \mid A_k \bar{x} > g_{ko}^+ \text{ and } A_k \text{ is a row of } R\}$$

$$IN = \{k \mid A_k \bar{x} < g_{ko}^- \text{ and } A_k \text{ is a row of } R\}$$

$$IM = \{k \mid g_{ko}^- \leq A_k \bar{x} \leq g_{ko}^+ \text{ and } A_k \text{ is a row of } R\}$$

and
$$IB = \{k \mid A_k \text{ is a row of } F \text{ or } A_k \text{ is a basic row}\}.$$

The algorithm will determine if a variation in a particular component of α, say α_p, will decrease the value of the objective function. All other α_i, for $i \neq p$, will remain at their current value. This perturbed vector is denoted by $\hat{\alpha}$. Its components will be $\hat{\alpha}_i = \alpha_i$ for $i \neq p$, and $\hat{\alpha}_p = \alpha_p + \epsilon$. If ϵ is chosen sufficiently small so that none of the elements $\bar{\delta}_k^*$ will move to their upper or lower bound, the rate of change of the objective function will be

$$\nabla z_p(\epsilon) = (w_{IB(p)}^* + \sum_{k \in IP} w_{k,s(k)}^+ A_k F_{\cdot p}^\# - \sum_{k \in IN} w_{k,s(k)}^- A_k F_{\cdot p}^\#)\,\text{sign}(\epsilon)$$

$$w_{IB(p)}^* = \begin{cases} w_{IB(p),\ell+1}^+ & \text{if } \alpha_p = g_{k\ell}^+ \equiv g_{IB(p),\ell}^+ \text{ and } \epsilon > 0 \\[2ex] w_{IB(p),\ell}^+ & \text{if } \alpha_p = g_{k\ell}^+ \qquad\qquad \text{and } \epsilon < 0 \\[2ex] w_{IB(p),\ell}^+ & \text{if } \alpha_p = g_{k\ell}^- \qquad\qquad \text{and } \epsilon > 0 \\[2ex] w_{IB(p),\ell+1}^- & \text{if } \alpha_p = g_{k\ell}^- \qquad\qquad \text{and } \epsilon < 0 \end{cases}$$

where we define $w_{IB(p),0}^+ = w_{IB(p),0}^- = 0$

and $F_{\cdot p}^\#$ is the p-th column of $F^\#$ and

$$\text{sign}(\epsilon) = \begin{cases} 1 & \epsilon > 0 \\ -1 & \epsilon < 0 \end{cases}$$

If the gradient $\nabla z_p(\varepsilon)$ is negative, determine how α_p can be varied before the rate of change becomes nonnegative. As $\hat{\alpha}_p$ varies, the gradient $\nabla z_p(\varepsilon)$ will change whenever any $\delta^*_{k,s(k)}$ reaches an upper or lower bound, and, therefore, it is necessary to recalculate $\nabla z_p(\varepsilon)$ at each of these new extreme points. This procedure of determining the new gradients will be further elaborated upon later in this section. At this point, the main aim is establish a procedure to find the sequence of the extreme points. The procedure is an extension of the standard minimum ratio test. From the system of nonbasic constraints, $RF^{\#}\alpha + I_R^-\delta_R^- - I_R^+\delta_R^+ + I_R\delta_R = g_{R0}$, it is recalled that there will be only one component, $\delta^*_{k,s(k)}$ for each constraint which is not at its upper or lower bound.

The first change in $\nabla z_p(\varepsilon)$ will occur for the minimum of the following ratios. The possible ratios are broken down into various cases and a brief explanation for each case is provided.

(1)
$$O^*(\Delta) = \underset{k}{\text{Minimum}}\left\{ \frac{\Delta g^*_{k,s(k)} - \delta^*_{k,s(k)}}{|A_k F^{\#}\cdot_p|} \; ; \; \text{sign}(\varepsilon)A_k F^{\#}\cdot_p > 0 \text{ and} \right.$$

$$k \in IP \text{ or}$$
$$\text{sign}(\varepsilon)A_k F^{\#}\cdot_p < 0 \text{ and}$$
$$k \in IN \cup IM$$

$\delta^*_{k\ell}$ increases to its upper bound

(2)
$$\phi^*(\Delta) = \underset{k}{\text{Minimum}}\left\{ \frac{\delta^{\#}_{k,s(k)}}{|A_k F^{\#}\cdot_p|} \; ; \; \text{sign}(\varepsilon)A_k F^{\#}\cdot_p < 0 \text{ and} \right.$$

$$k \in IP \text{ or}$$
$$\text{sign}(\varepsilon)A_k F^{\#}\cdot_p > 0 \text{ and}$$
$$k \in IN \cup IM$$

$\delta^*_{k\ell}$ decreases to zero

$$(3) \qquad \Theta^* = \begin{cases} |g^*_{IB(p),\ell+1} - g^*_{IB(p),\ell}|, & \varepsilon > 0 \text{ and } * = + \\ & \text{or} \\ & \varepsilon < 0 \text{ and } * = - \\ |g^*_{IB(p),\ell} - g^*_{IB(p),\ell-1}| & \varepsilon < 0 \text{ and } * = + \\ & \text{or} \\ & \varepsilon > 0 \text{ and } * = - \end{cases}$$

where we define $g^-_{IB(p),-1} = g^+_{IB(p),0}$ and $g^+_{IB(p),-1} = g^-_{IB(p),0}$

This is the possible change in α_p before $\delta^*_{IB(p),\ell}$ reaches an upper bound or $\delta^*_{IB(p),\ell-1}$ reaches a lower bound. Therefore, the first extreme point along the path will be associated with the minimum ratio:

$$\Theta = \text{MIN } (\Theta^*(\Delta), \phi^*(\Delta), \Theta^*)$$

The algorithm now updates the gradient for this new extreme point. The description of this process is also broken down into the cases which correspond to those of the minimum ratio. The following indicates the term to be added to $\nabla z_p(\varepsilon)$.

$$(1) \qquad (w^*_{u,s(k)+1} - w^*_{u,s(k)})|A_u F^\#_{\cdot p}| \qquad\qquad u \in IP$$

$$(2) \qquad (w^*_{u,s(k)} - w^*_{u,s(k)-1})|A_u F^\#_{\cdot p}|$$

$$(3) \qquad w^*_{IB(p),\ell+1} - w^*_{IB(p),\ell}$$

and

$$w^*_{IB(p),\ell} - w^*_{IB(p),\ell-1}$$

where we define $w_{k0} = w^-_{k0} = w^+_{k0} = 0,\quad w^+_{k,-1} = w_{k1} = w^-_{k1}$

and $w^-_{k,-1} = w^+_{k1}$

where u corresponds to the row yielding the minimum ratio.

When the new gradient has been calculated, then we shall either proceed to the next extreme point on the path it is has remained negative, or we shall perform the standard simplex pivot operation if it has become nonnegative. If the pivot is performed, then IB(p) will leave the basis and be replaced by the equation in R associated with the minimum ratio.

If the new rate of change is still negative, the algorithm updates the particular ratio which provided the minimum. All other ratios will remain the same. To provide simplicity in the exposition and also to illuminate the process, we shall assume that the minimum ratio was associated with case (1) and that row r yielded this ratio. Thus, the distance to the next extreme point (or goal-level) for the equation associated with this minimum ratio can be found by adding the width of the next goal-segment to the numerator of the ratio. This new ratio is denoted as follows:

$$\Theta^{+}(\Delta + \Delta_1) = \frac{(\Delta g_{r\ell}^{+} - \bar{\delta}_{r\ell}^{+}) + g_{r_1}^{+}q(r)}{|A_{kp}F^{\#}|}; \quad sign(\varepsilon)*A_{kp}F^{\#} > 0$$

The new ratio $\Theta^{+}(\Delta + \Delta_1)$ replaces the ratio in the r-th row, and the new minimum ratio is determined. The rate of change is then updated. This process is repeated until the rate of change becomes nonnegative, and at that point the pivot procedure is implemented, if the minimum ratio is not given by Θ_3.

An additional aspect of the algorithm is that some bookkeeping is required to accomodate the movement of $\delta_{k\ell}$ for cases (2) and (4), respectively. If the multiple pivoting technique proceeds along an

extreme point path starting at $\delta_{k\ell}^+$, for example, then the variables associated with the decreasing sequence of segments would be $\delta_{k(\ell-1)}^+$, $\delta_{k(\ell-2)}^-$,....., and, δ_k^+. If further decreases in the objective function are possible, then pass into the goal-interval. Thus, as the algorithm through the various types of intervals associated with the index sets IP, IN, and IM, this process must be monitored.

Application of this pivoting procedure will result in the optimal solution being obtained in a finite number of iterations (degeneracy being resolved in an appropriate manner). The algorithm increases, or decreases, the value of a "basic" goal constraint until any further unilateral change would increase the value of the goal functional. The end result of this process is to, at times, combine several standard iterations into a single one. Thus, basis updates and the recalculation of ratios are reduced.

4. Conclusions

Goal programming has long been recognized as one of the most popular modeling techinques in operations research. Despite the many applications of goal programming, little research seems to have been directed towards special algorithms to solve the resulting problems. One recent effort in this area is given by Charnes, Cooper, Klingman and Niehaus [12]. In this paper, we have developed a specialized algorithm to solve goal-interval programming problems. The algorithm is able to exploit the unique structure of problems formulated in this manner. It possesses a capability to combine several standard simplex pivots into a single pivot. Computational studies with various computer code implementations of the algorithm will be the subject of future research.

While the main thrust of this paper has been devoted to algorithmic considerations, the importance of the model itself should not be overlooked. Many goal programming problems found in the literature can be expressed in the framework of (3.1)-(3.5). Refinement of the goal intervals will also allows us to obtain an ϵ-optimal solution to a separable nonlinear goal functional. An immediate consequence of this is to allow consideration of the L_p norm $1 \leq p \leq \infty$.

References:

1. N.N. Abdelmalek, N. N., "An Efficient Method for the Discrete Linear L_1 Approximation Problem," Mathematics of Computation, 29(1975), 844-850.

2. R. D. Armstrong and E. L. Frome, "A Comparison of Two Algorithms for Absolute Deviation Curve Fitting," Journal of the American Statistical Association, 71(1976), 328-330.

3. R. D. Armstrong and J. W. Hultz, "A Restricted Discrete Approximation Problem in the L_1 Norm," SIAM Journal of Numerical Analysis, 14(June, 1977), 555-565.

4. I. Barrodale and F. D. K. Roberts, "An Improved Algorithm for Discrete L_1 Linear Approximation," SIAM Journal of Numerical Analysis, 10(1973), 839-848.

5. I. Barrodale and A. Young, "Algorithms for Best L_1 and L_∞ Linear Approximations on a Discrete Set," Numerische Mathematique, 8(1966), 295-306.

6. A. Ben-Israel and A. Charnes, "An Explicit Solution of a Special Class of Linear Programming Problems," Operations Research, 16(1968), 1166-1175.

7. A. Charnes, "Optimality and Degeneracy in Linear Programming," Econometrica, 20(1952), 160-170.

8. A. Charnes and B. Ccllomb, "Optimal Economic Stabilization Policy: Linear: Linear Goal-Interval Programming Models," Journal Social Economic Planning Science, 6(1972), 431-435.

9. A. Charnes and W. W. Cooper, "Goal Programming and Multiple Objective Optimations," European Journal of Operations Research, 1(1977), 39-54.

10. A. Charnes and W. W. Cooper, Management Models and Industrial Applications of Linear Programming, John Wiley & Sons, Inc., New York, 1961.

11. A. Charnes, W. W. Cooper, and R. O. Ferguson, "Optimal Estimation of Executive Compensation by Linear Programming," Management Science, 1(1955), 138-151.

12. A. Charnes, W. W. Cooper, D. Klingman, and R. J. Niehaus, "Explicit Solutions to Convex Goal Programming," Management Science, 22(1975), 438-448.

13. A. Charnes, W. W. Cooper, and R. J. Niehaus, "Studies in Manpower Planning," U.S. Navy Office of Civilian Manpower Management, Washington, 1972.

14. A. Charnes and C. E. Lemke, "Minimization of Nonlinear Separable Convex Functionals," Naval Research Logistics Quarterly, 1(1954), 301-312.

15. B. P. Collomb, "Goal Interval Approaches to Intertemporal Analysis and Decentralization in Management," Ph.D. Thesis, The University of Texas, Austin, Texas, 1971.

16. K. Spyropoulos, E. Kiountouzis, and A. Young, "Discrete Approximations in the L_1 Norm," Computer Journal, 16(1973), 180-186.
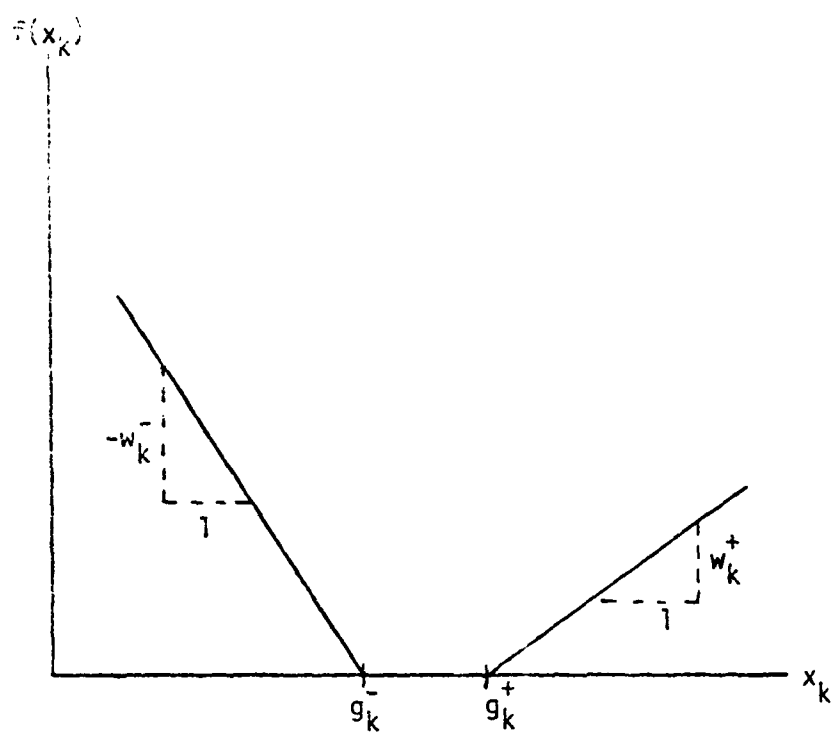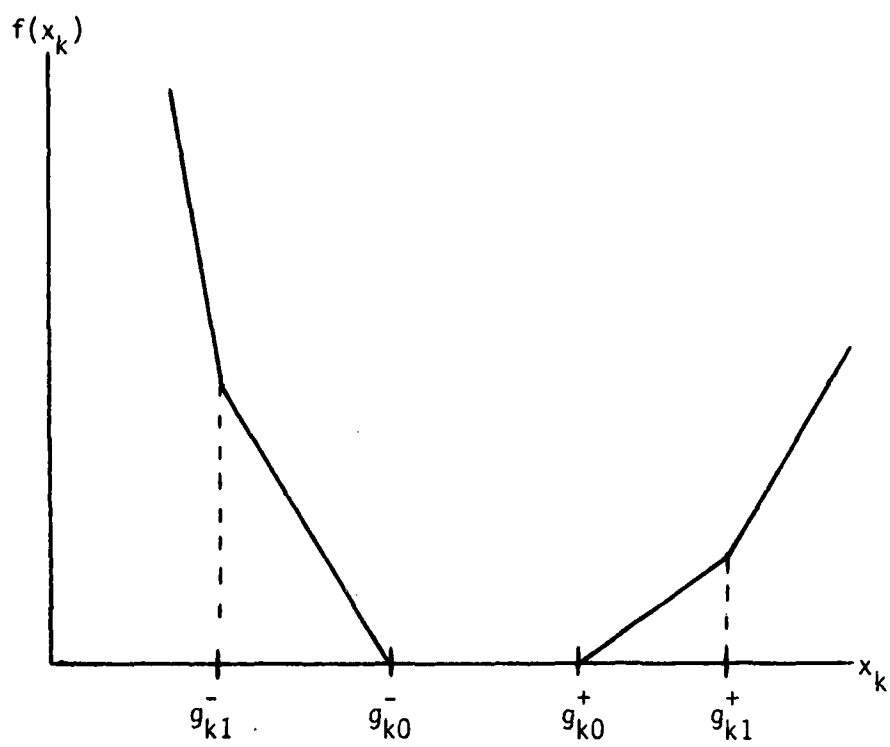
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13. ABSTRACT

Goal programming has become a popular and important planning tool in many areas. However, the main direction of goal programming research has been in formulating models instead of seeking procedures that would provide improved computational efficiency.

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